Hecke insertion and maximal increasing and decreasing sequences in fillings of polyominoes

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Triangle Lectures in Combinatorics
University of North Carolina at Charlotte

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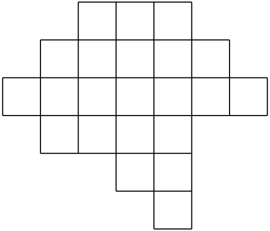
Moon Polyominoes

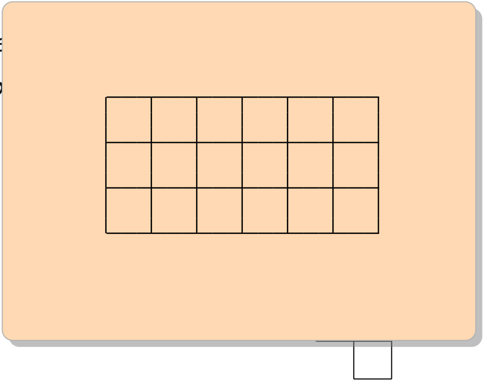
- *convex* rows and columns
- *comparable* rows and columns



- Lengths of rows from top to bottom form a unimodal sequence.

Moon Polyominoes

- *conve*
- *comp*



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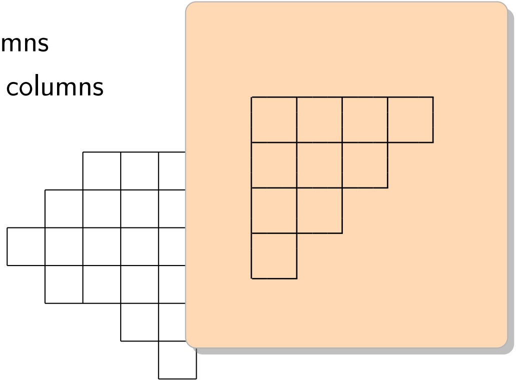

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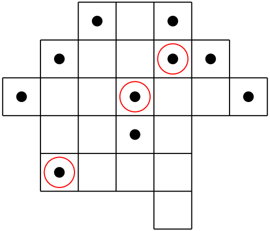
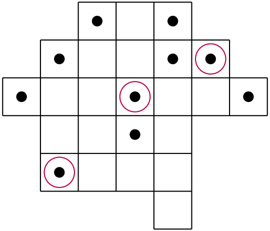
Chains in 01-fillings



- ne-chain: A set of 1-cells $\{(i_1, j_1), (i_2, j_2), \ldots, (i_k, j_k)\}$ with $i_1 < \cdots < i_k, j_1 < \cdots < j_k$ such that the smallest rectangle containing them is a subset of the polyomino.
- $\mathrm{ne}(M) =$ the length of the largest ne-chain in the filling M.

Chains in 01-fillings



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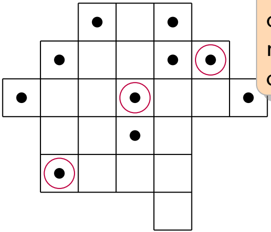

Not an ne-chain because the rectangle is not part of the polyomino

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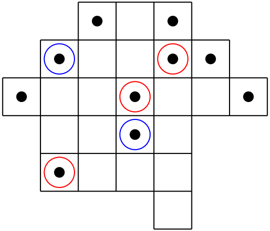
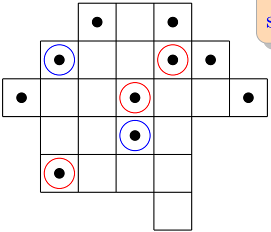


- se-chain: A set of 1-cells $\{(i_1, j_1), (i_2, j_2), \ldots, (i_k, j_k)\}$ with $i_1 < \cdots < i_k, j_1 > \cdots > j_k$ such that the smallest rectangle containing them is a subset of the polyomino.
- $\mathrm{se}(M)$ = the length of the largest se-chain in the filling M.

Chains in 01-fillings



$\mathrm{ne}(M) = 3$
$\mathrm{se}(M) = 2$

- se-chain: A set of 1-cells $\{(i_1, j_1), (i_2, j_2), \ldots, (i_k, j_k)\}$ with $i_1 < \cdots < i_k, j_1 > \cdots > j_k$ such that the smallest rectangle containing them is a subset of the polyomino.
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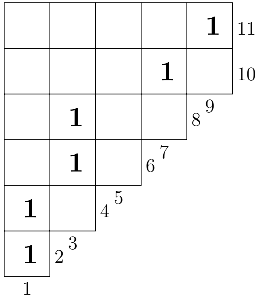
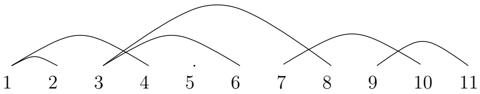
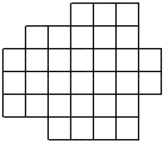


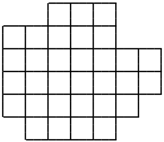


- permutations and words
- graphs
- set partitions, linked partitions, matchings
- crossings and nestings of edges in the picture

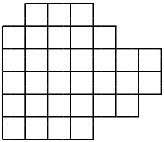
Restriction on columns: Exactly one 1 per column



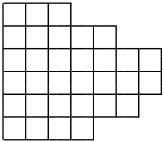
a. $\mathcal{M}_1$



b. $\mathcal{M}_2$

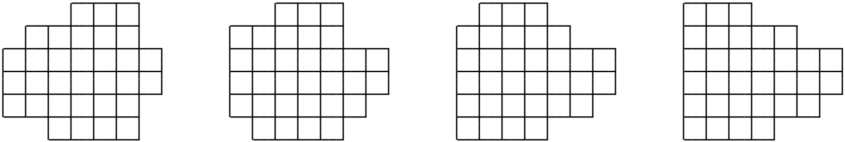


c. $\mathcal{M}_3$



d. $\mathcal{M}_4$

Restriction on columns: Exactly one 1 per column



a. $\mathcal{M}_1$ b. $\mathcal{M}_2$ c. $\mathcal{M}_3$ d. $\mathcal{M}_4$

$$G_i(x,y) := \sum_{M \in \mathcal{F}(\mathcal{M}_i)} x^{\mathrm{ne}(M)} y^{\mathrm{se}(M)}$$

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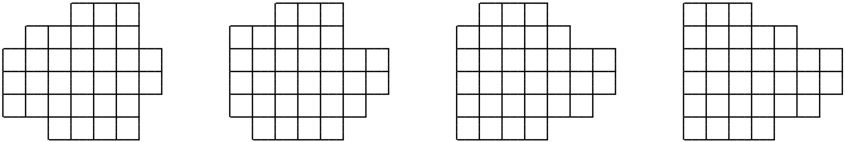

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$$G_1(x,y) = 40x^3y^3 + 238(x^3y^2 + x^2y^3) + 4(x^3y + xy^3) + 348x^2y^2 + 2(x^2y + xy^2)$$

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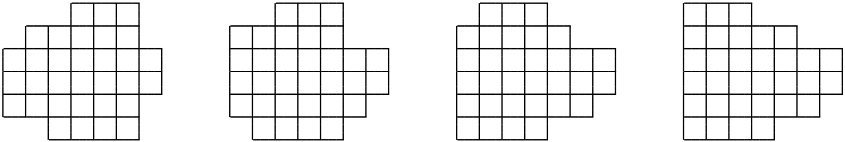

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$$G_2(x,y) = G_3(x,y) = G_4(x,y) = G_1(x,y)$$

Conjectures for moon polyominoes

Conjecture

For a moon polyomino $\mathcal{M}$, if

$$G(x,y) = \sum_{M \in \mathcal{F}(\mathcal{M})} x^{\mathrm{ne}(M)} y^{\mathrm{se}(M)}$$

then

$$G(x,y) = G(y,x).$$

Conjecture

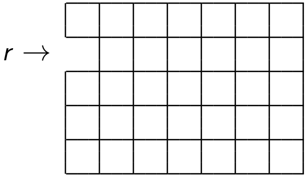
If $\mathcal{M}'$ is obtained by permuting the rows and/or columns of $\mathcal{M}$, then

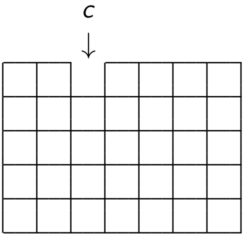
$$\sum_{M \in \mathcal{F}(\mathcal{M}')} x^{\mathrm{ne}(M)} y^{\mathrm{se}(M)} = \sum_{M \in \mathcal{F}(\mathcal{M})} x^{\mathrm{ne}(M)} y^{\mathrm{se}(M)}.$$

Previous work

- Chen, Deng, Du, Stanley, Yan (2007) - set partitions
- Backelin, West , Xin (2007) + Krattenthaler (2006) + de Mier (2006) - Ferrers shapes
- Jonsson (2007) + Jonsson and Welker (2007) - stack polyominoes
- Rubey (2011) - moon polyominoes for ne-chains
- Poznanović and Yan (2014) - almost moon polyominoes for ne-chains

Almost-moon: one exception to the convexity rule





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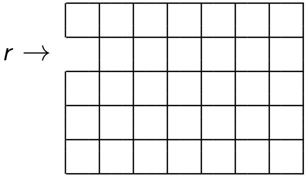
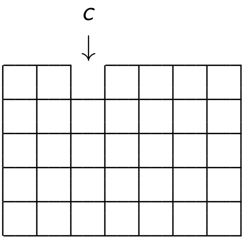
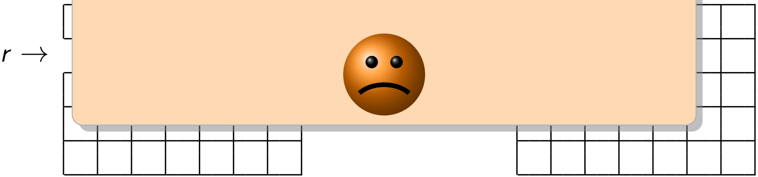




$$
\begin{aligned}
G_1(x,y) = {} & (15x^5y^3 + 13x^3y^5) + 56x^4y^4 + (80x^5y^2 + 82x^2y^5) + (1180x^4y^3 + 1178x^3y^4) \\
& + 5(x^5y + xy^5) + (1210x^4y^2 + 1212x^2y^4) + 5370x^3y^3 + 10(x^4y + xy^4) \\
& + (1477x^3y^2 + 1473x^2y^3) + 64x^2y^2. \\
G_2(x,y) = {} & (8x^5y^3 + 15x^3y^5) + 48x^4y^4 + (83x^5y^2 + 77x^2y^5) + (1129x^4y^3 + 1174x^3y^4) \\
& + (9x^5y + 8xy^5) + (1273x^4y^2 + 1227x^2y^4) + 5434x^3y^3 + (6x^4y + 7xy^4) \\
& + (1415x^3y^2 + 1467x^2y^3) + 60x^2y^2.
\end{aligned}
$$

Almost-moon: one exception to the convexity rule

The idea of applying local changes to change the shape doesn't work for our conjecture.



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Another idea: Hecke insertion

word $w \longleftrightarrow$ pair of tableaux $(P(w), Q(w))$ of same shape

(Buch, Kresch, Shimozono, Tamvakis, Alexander Yong, 2008)

- $P(w)$ is an increasing tableau: across rows and down columns, possibly repeated entries
- $Q(w)$ is a set-valued tableau: multiple entries per cell, increasing across rows and down columns

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For $w = 32412143$:

$$P(w) = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 2 & 4 \\ \cline{1-2} 3 \\ \cline{1-1}\end{array} \qquad Q(w) = \begin{array}{|c|c|c|}\hline 1 & 3 & 7 \\ \hline 2 & 5,8 \\ \cline{1-2} 4,6 \\ \cline{1-1}\end{array}$$

What's the connection?

- $\mathrm{lis}(w)$ = length of the longest increasing subsequence of w
- $\mathrm{lds}(w)$ = length of the longest decreasing subsequence of w

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Theorem (Thomas, Yong, 2011)

For a word w, let $P(w)$ be the Hecke insertion tableau of w. Then

- $\mathrm{lis}(w) = \#$ *columns of* $P(w)$
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Hecke insertion: to construct P

To Hecke insert an integer x into a tableau Y:

- x is inserted in the first row R; if an output integer y is produced, then it's inserted in the next row, etc.
- If x is larger than or equal to all entries in R, then there are 2 possibilities:
  - If adding x as a new box to the first row results in an increasing tableau, do that and stop.
  - Otherwise, stop.
- Otherwise, x is strictly smaller than some entry in the first row. Let y be the smallest integer in R that is strictly larger than x.
  - If replacing y with x results in an increasing tableau, then replace y with x and insert y into the next row.
  - If replacing y with x does not result in an increasing tableau, then insert y into the next row and do not change R.

Hecke insertion: to construct $P(w)$ and $Q(w)$

1	2	4	6
3	4		
5	7		
6			

Y

1	2	4	6
3	4		
5	**7** $\longleftarrow c$		
6			

$Z = Y \overset{\mathrm{H}}{\longleftarrow} 2$

1	2	4	5
3	4	**6** $\longleftarrow c$	
5	7		
6			

$Z = Y \overset{\mathrm{H}}{\longleftarrow} 5$

- c is at the bottom of the column of Z containing the rightmost box of the row in which the algorithm stops.
- $P(w) = (\cdots((\emptyset \overset{\mathrm{H}}{\longleftarrow} w_1) \overset{\mathrm{H}}{\longleftarrow} w_2) \cdots \overset{\mathrm{H}}{\longleftarrow} w_n)$
- $Q(w)$ is built by inserting k in the box c that gets recorded when w_k is inserted.

Growth diagrams (Patrias, Pylyavskyy 2018)

The Hecke growth diagram of the word 213312.

$$
\begin{array}{ccccccccccccc}
\emptyset & \text{---} & 1 & \text{---} & 11 & \text{---} & 21 & \overset{1}{\text{---}} & 21 & \overset{2}{\text{---}} & 21 & \text{---} & 22 \\
| & & | & & | & X & | & X & | & & | & & | \\
\emptyset & \text{---} & 1 & \text{---} & 11 & \text{---} & 11 & \text{---} & 11 & \overset{2}{\text{---}} & 11 & \text{---} & 21 \\
| & X & | & & | & & | & & | & & | & X & | \\
\emptyset & \text{---} & \emptyset & \text{---} & 1 & \text{---} & 1 & \text{---} & 1 & \overset{1}{\text{---}} & 1 & \text{---} & 1 \\
| & & | & X & | & & | & & | & X & | & & | \\
\emptyset & \text{---} & \emptyset & \text{---} & \emptyset & \text{---} & \emptyset & \text{---} & \emptyset & \text{---} & \emptyset & \text{---} & \emptyset
\end{array}
$$

- Local rules are applied to grow the diagram from bottom left to top right: corners labeled by partitions, some horizontal edges by integers

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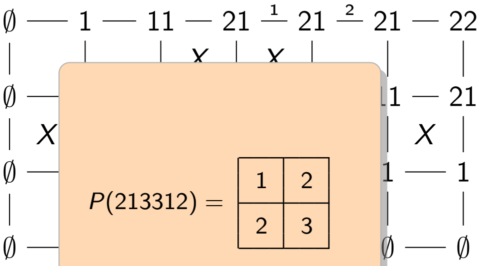
| & X & | & & | & & | & & | & & | & X & | \\
\emptyset & \text{---} & \emptyset & \text{---} & 1 & \text{---} & 1 & \text{---} & 1 & \overset{1}{\text{---}} & 1 & \text{---} & 1 \\
| & & | & X & | & & | & & | & X & | & & | \\
\emptyset & \text{---} & \emptyset & \text{---} & \emptyset & \text{---} & \emptyset & \text{---} & \emptyset & \text{---} & \emptyset & \text{---} & \emptyset
\end{array}
$$

- Local rules are applied to grow the diagram from bottom left to top right: corners labeled by partitions, some horizontal edges by integers
- $P(w)$ is on the right boundary
- $Q(w)$ is on the top boundary

The Hecke growth diagram of the word 213312.



$P(213312) = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 2 & 3 \\ \hline \end{array}$

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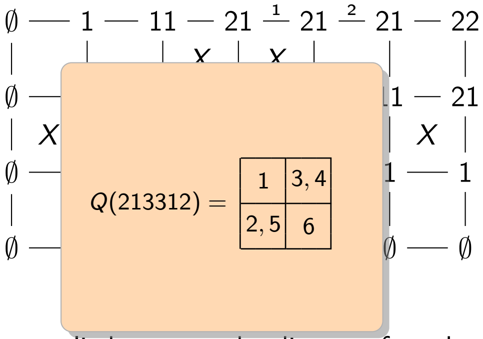

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K-jeu de taquin: $\mathrm{jdt}_C(T)$

(Thomas, Yong, 2009)

- T is an increasing skew tableau
- C = set of inner corners of T (on the upper left side)
- In step k: if k is directly next to a $\bullet$, swap k and $\bullet$

	$\bullet$	2
$\bullet$	2	3
1	3	

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1	3	

	$\bullet$	2
1	2	3
$\bullet$	3	

Swap $\bullet$ and 1

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1	2	3
$\bullet$	3	

	2	$\bullet$
1	$\bullet$	3
$\bullet$	3	

Swap $\bullet$ and 2

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1	2	3
$\bullet$	3	

	2	$\bullet$
1	$\bullet$	3
$\bullet$	3	

	2	3
1	3	$\bullet$
3	$\bullet$	

Swap $\bullet$ and 3

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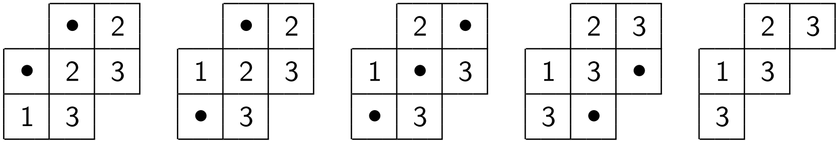

$\mathrm{jdt}_C(T)$

K-Knuth equivalence

Definition

Two words are said to be *K-Knuth equivalent* if one can be obtained from the other via a finite series of applications of the following K-Knuth relations:

$$
\begin{aligned}
xzy &\equiv zxy \qquad (x < y < z)\\
yxz &\equiv yzx \qquad (x < y < z)\\
x &\equiv xx\\
xyx &\equiv yxy.
\end{aligned}
$$

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Two words are said to be *K-Knuth equivalent* if one can be obtained from the other via a finite series of applications of the following K-Knuth relations:

$$\begin{aligned} xzy &\equiv zxy \qquad (x < y < z) \\ yxz &\equiv yzx \qquad (x < y < z) \\ x &\equiv xx \\ xyx &\equiv yxy. \end{aligned}$$

- Words that have the same Hecke insertion tableau P are K-Knuth equivalent.

Properties linking all this

Recall

- $\mathrm{lis}(w)$ = length of the longest increasing subsequence of w
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Theorem (Thomas, Yong, '09)

If $w_1 \equiv w_2$ *then* $\mathrm{lis}(w_1) = \mathrm{lis}(w_2)$ *and* $\mathrm{lds}(w_1) = \mathrm{lds}(w_2)$.

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Theorem (Buch, Samuel, '16)

Let $[a, b]$ be an integer interval. Let $w_1 \equiv w_2$. For $i = 1, 2$, let $w_i|_{[a,b]}$ be the word obtained from w_i by deleting all integers not contained in the interval $[a, b]$. Then $w_1|_{[a,b]} \equiv w_2|_{[a,b]}$.

Properties linking all this

For an increasing tableau T, $\mathrm{row}(T)$ is the reading word of T, obtained by reading the entries of T from left to right along each row, starting from the bottom row and moving upward.

$$T = \begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 2 & 4 \\ \cline{1-2} 3 \\ \cline{1-1} \end{array}$$

$$\mathrm{row}(T) = 324123$$

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$$w \equiv \mathrm{row}(P(w))$$

For example,

$$P(32412143) = T \implies 32412143 \equiv 324123$$

Theorem (Buch, Samuel, '16)

Let T and T' be increasing tableaux. Then $\mathrm{row}(T) \equiv \mathrm{row}(T')$ if and only if T and T' are K-jeu de taquin equivalent.

Theorem (Chen, Guo, Pang, '15)

Let w be a word of positive integers, and k be the maximal element appearing in w. Let w' be the word obtained from w by deleting the elements equal to k. Then $P(w')$ is obtained from $P(w)$ by deleting the squares occupied with k.

What we can prove

Theorem

If $\mathcal{M}$ and $\mathcal{M}'$ are stack polyominos with same row lengths then

$$G_{\mathcal{M}'}(x, y) = G_{\mathcal{M}}(x, y).$$

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$$G_{\mathcal{M}'}(x, y) = G_{\mathcal{M}}(x, y).$$

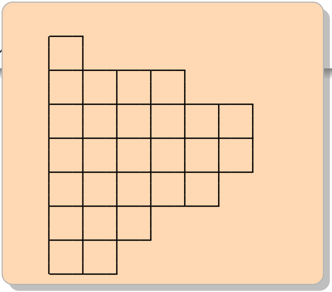
$$G_{\mathcal{M}}(x, y) = \sum_{M \in \mathcal{F}(\mathcal{M})} x^{\mathrm{ne}(M)} y^{\mathrm{se}(M)}$$

What we can prove

Theorem

If $\mathcal{M}$ and $\mathcal{M}'$ are stack polyominos with same row lengths then

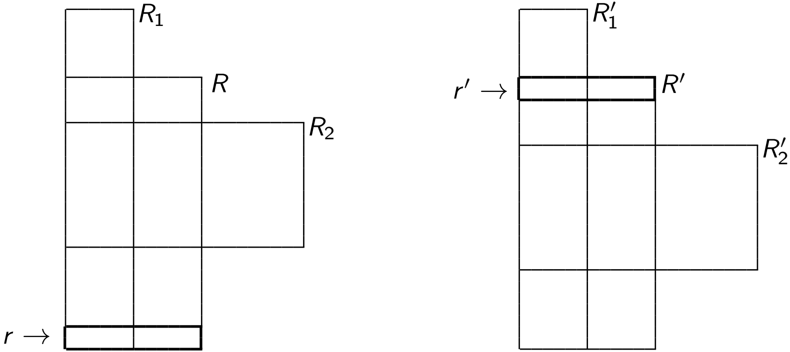
$$G_{\mathcal{M}}$$



Approach: Bijection between fillings of very similar shapes

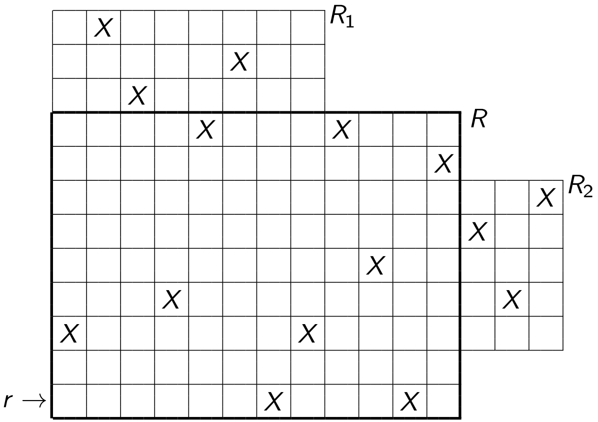
Theorem

Let $\mathcal{M}$ be a stack polyomino and let $\mathcal{M}'$ be obtained by moving the bottom row of $\mathcal{M}$ up. Then $G_{\mathcal{M}'}(x,y) = G_{\mathcal{M}}(x,y)$.



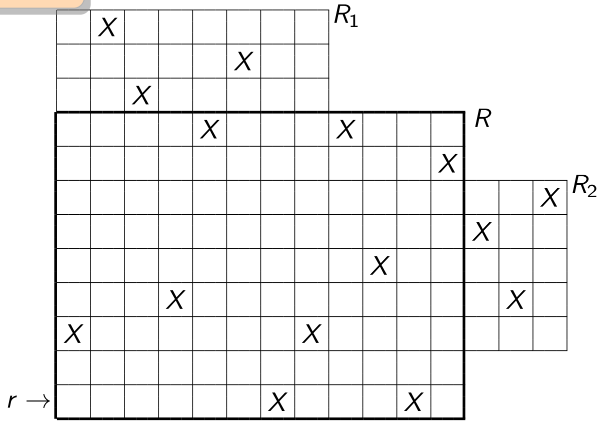
The bijection

Everything outside of R stays the same.



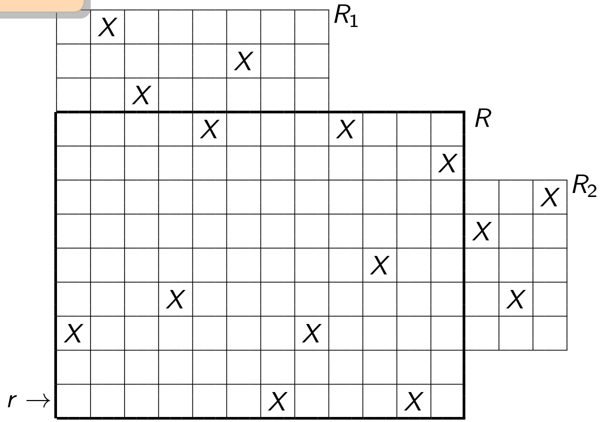
The bijection

Empty rows and columns in R stay empty.



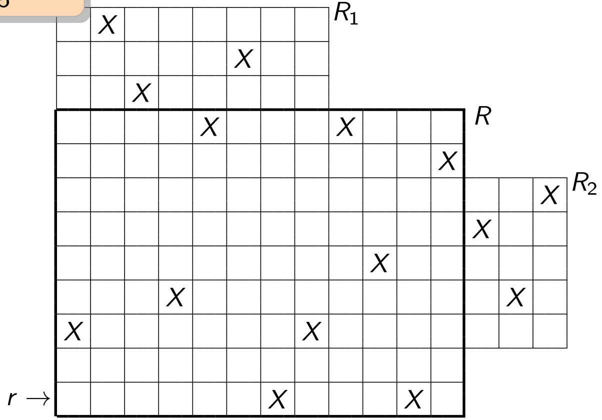
The bijection

Let w be the word from R.



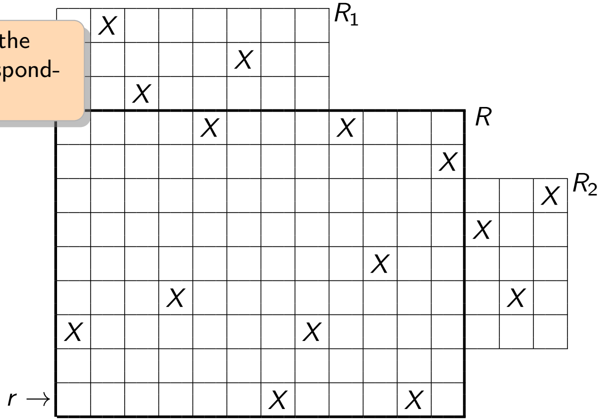
The bijection

$w = 236126415$

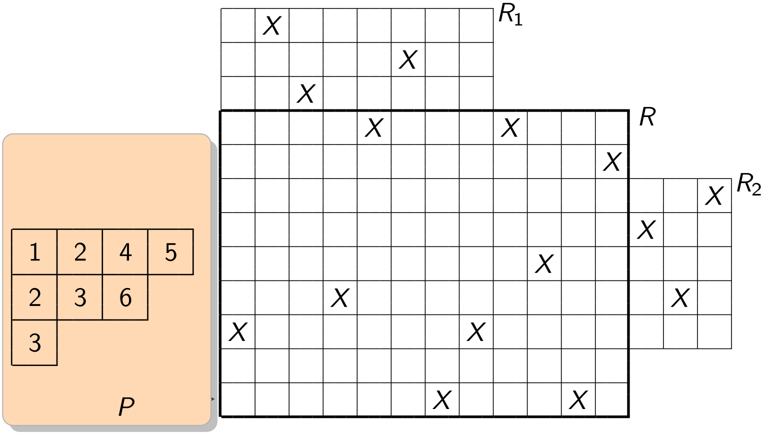


The bijection

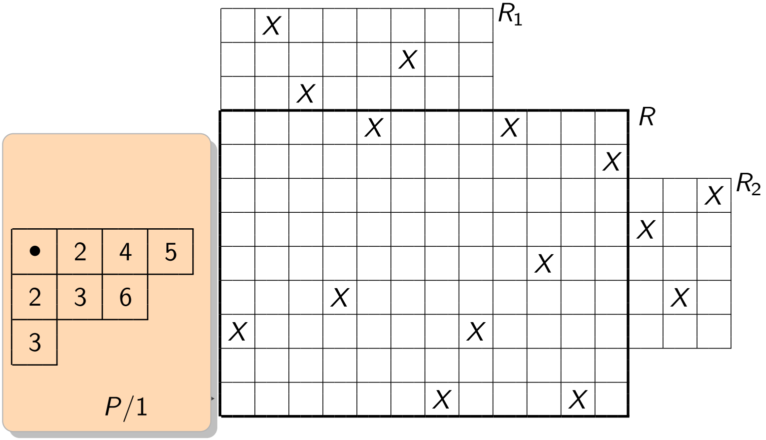
Let (P, Q) be the tableaux corresponding to w.

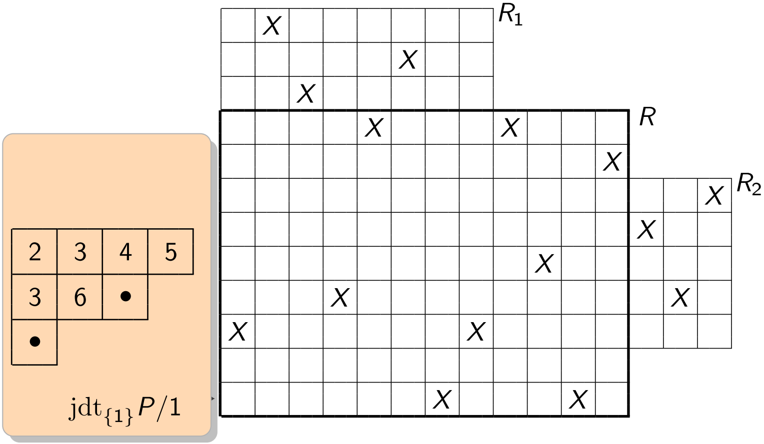


The bijection

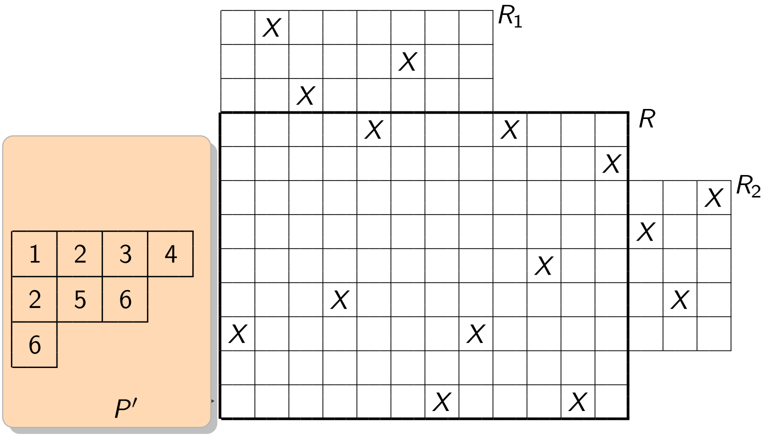


The bijection



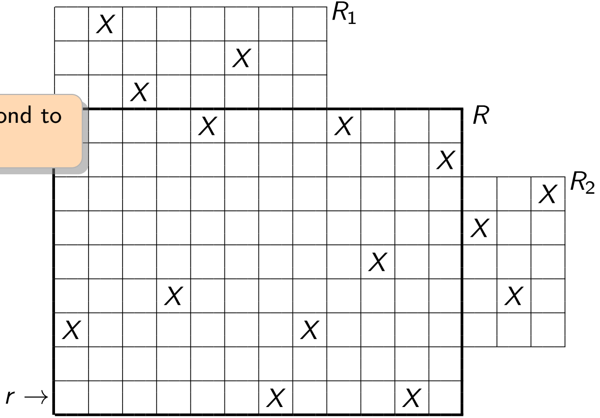

R_1
R
R_2
2 3 4 5
3 6 •
•
$\mathrm{jdt}_{\{1\}}P/1$

The bijection

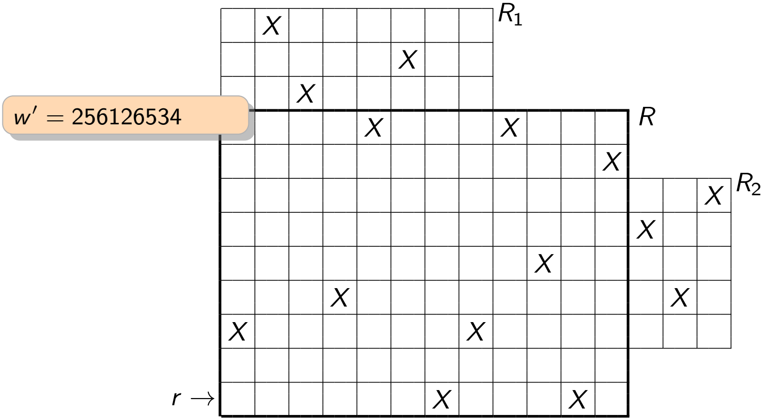


The bijection

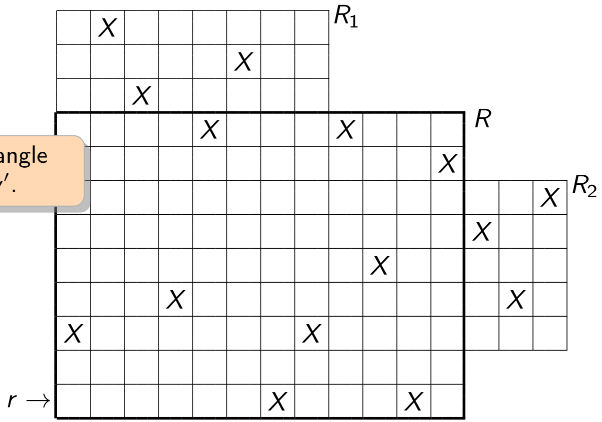
Let w' correspond to (P', Q).



The bijection

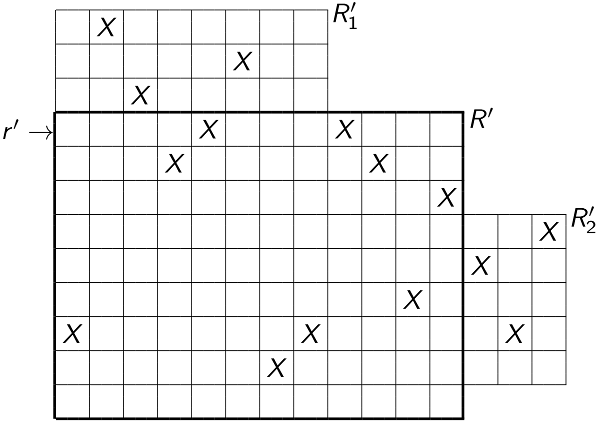


The bijection



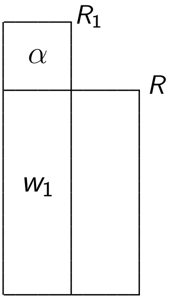
Fill in the rectangle R' in $\mathcal{M}'$ by w'.

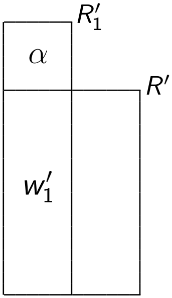
The resulting filling of $\mathcal{M}'$



ne and se are preserved

- w and w' have Hecke insertion tableaux of same shape, so $\mathrm{lis}(w') = \mathrm{lis}(w)$ and $\mathrm{lds}(w') = \mathrm{lds}(w)$.
- The fillings in R_1 and R_1' have the same recording set valued tableau.





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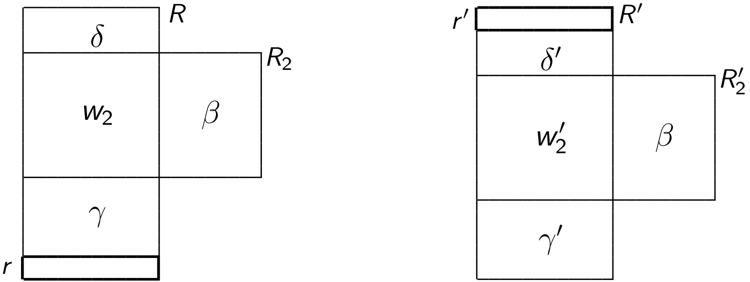


$$w \backslash 1 \equiv \mathrm{row}(P) \backslash 1 = \mathrm{row}(P/1) \equiv \mathrm{row}(P'/m) = \mathrm{row}(P') \backslash m \equiv w' \backslash m$$

$$\gamma + w_2 + \delta \equiv \gamma' + w_2' + \delta'$$

$$w_2 \equiv w_2'$$

$w_2\beta \equiv w_2'\beta$ so $\mathrm{lis}(w_2\beta) = \mathrm{lis}(w_2'\beta)$ and $\mathrm{lds}(w_2\beta) = \mathrm{lds}(w_2'\beta)$

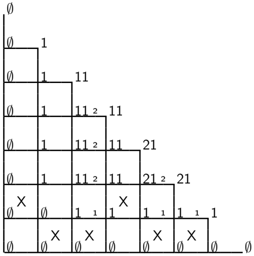
Remarks

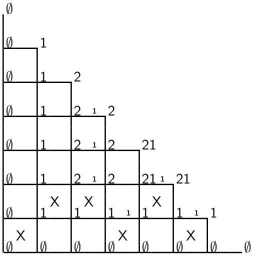
- Moving an arbitrary row up in the same way doesn't work.
- This construction doesn't preserve $(\mathrm{ne}, \mathrm{se})$ for non-stack polyominoes

The conjecture for general moon polyominoes is still open!

Maximal crossings and nestings in linked partitions

(Chen, Guo, Pang, '15)





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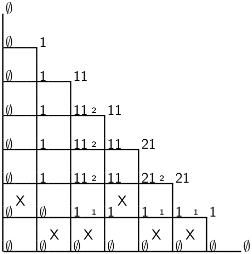
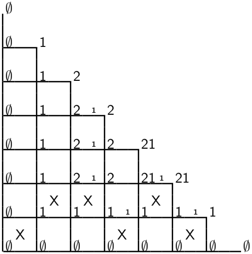
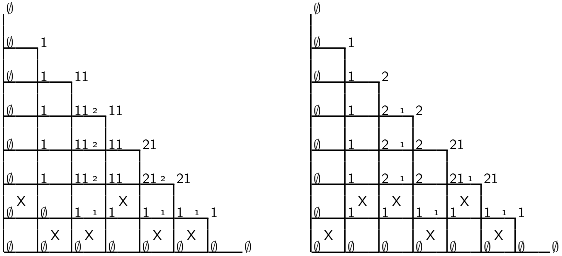




- Reflect the first tableau about the x-axis

(Chen, Guo, Pang, '15)



- Reflect the first tableau about the x-axis
- Apply the 'move bottom row up' rule several times

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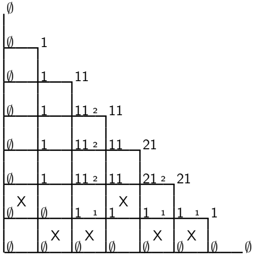
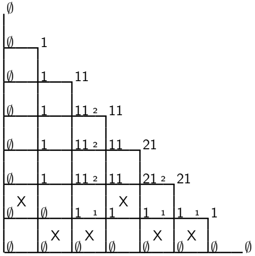
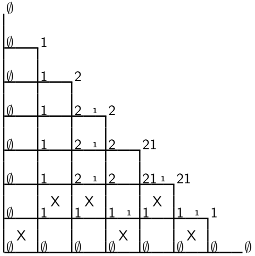


- Reflect the first tableau about the *x*-axis
- Apply the 'move bottom row up' rule several times
- This produces a filling of the same initial shape with $(\mathrm{ne}, \mathrm{se})$ reversed

(Chen, Guo, Pang, '15)





- Reflect the first tableau about the x-axis
- Apply the 'move bottom row up' rule several times
- This produces a filling of the same initial shape with $(\mathrm{ne}, \mathrm{se})$ reversed
- This is a different bijection from the one described above

Thank you.