Problems of the month, October 2024

Problem 1. Consider a convex polyhedron for which every face is a triangle. The vertices of this polyhedron are colored using 3 colors. Show that the number of faces of the polyhedron whose vertices consist of all three colors is even.

Problem 2. Consider a function $f : \mathbb{R}^2 \to \mathbb{R}$ which has continuous partial derivatives. Suppose that $f(x, 0) = 0$ for all x and that $(\partial_x f(x, y))^2 + (\partial_y f(x, y))^2 \leq \partial_x f(x, y)$ for all x, y. Show that $f(x, y) = 0$ for all x, y.

First correct solution for either problem from an undergraduate this month wins a CASH PRIZE!